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Energy-speed relation and optimal speed during level walking*

By

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With 4 Figures in the Text

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In studying the energy expenditure of physically disabled human subjects during walking, it became necessary in this laboratory, for comparative purposes, to determine normal values of energy expenditure at various speeds. There is a large literature dealing with this subject, but no completely satisfactory quantitative relationships between energy expenditure and speed of walking in normal subjects have been established.

PASSMORE and DURNIN (1955), in their review of human energy expenditure, compiled data of earlier investigators and found that energy expenditure could be represented as a linear function of speed in the range 50—100 meters/min. This relationship, while of considerable practical usefulness, is not very satisfying since it holds only for restricted body weight and does not lend itself to any theoretical deductions.

COTES, MEADE and WISE (1957) have developed an equation in which energy expenditure during walking is expressed as a linear function of lift-work. This equation adequately predicts energy expenditure for speeds up to about 110 meters/min. The equation is not satisfactory for much higher speeds, and its usefulness for certain deductive purposes is limited by its rather complex mathematical form and by the considerable number of variables which it contains.

It is the purpose of this paper to establish a mathematical relationship between energy expenditure and speed which is useful in the sense that it is simple and of suggestive physical form, having adequate predictive value and lending itself to deductions of other interesting features of energy expenditure during walking.

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Method

Twelve male and 7 female normal untrained adults were studied. A summary of the physical characteristics of the subjects is given in Table 1. All body weights listed in Table 1 and all weights used in calculations in this paper refer to the lightly clothed subject.

The subjects walked around a rectangular track in the laboratory, 24.4 meters long. Most measurements were made at speeds of 24.4, 48.8, 73.2 and 97.6 meters/min. A few measurements at higher speeds, up to 140 meters/min., were also made. Test runs were usually of 10 minutes duration. Room temperatures varied from 18^0 to 26^0 C, with a mean of 23^0 C.

Table 1. *Physical characteristics of subjects*

	Males	Females
Age range, yr	22—51	22—45
Mean age, yr	32	32
Height range, cm	168—186	153—168
Mean height, cm	179	162
Weight range, kg	62—90	48—63
Mean weight, kg	76	54

Respiratory minute-volumes were determined with a Max Planck-type respirometer (MÜLLER and FRANZ, 1952) weighing 3.5 kg, carried on the back. Oxygen concentrations of expired air were measured with a Beckman-Pauling oxygen analyzer. Energy equivalents were calculated by the method of WEIR (1949).

In agreement with the findings of MONTOYE, HUSS, REINEKE and COCKRELL (1958), MAX PLANCK respirometers were found to underestimate gas flow unless corrected upward by a variable amount, usually of the order of 4—8%. So corrected, these respirometers proved to be rugged and reliable instruments for measuring minute-volumes up to about 50 l/min.

Results and Discussion

The following symbols will be used: $\dot{E}$, Kcal/min; $\dot{E}_w$, cal/min/kg; E_m, cal/meter/kg; v, speed in meters/min; w, body weight in kg; R, leg length in meters; *d*, foot-length in meters; *n*, steps/min.

1. The Linear Relationship between $\dot{E}_w$ and v^2. PASSMORE and DURNIN (1955) showed that the results from several different laboratories agreed closely in the curve relating $\dot{E}$ to *v*. This curve strongly suggests a parabola of the form $\dot{E} = b + mv^2$, where *b* and *m* are constants. It would be very satisfying if the relation between energy expenditure and speed should include a v^2 term, since energy expenditure should be some function of one or more kinetic energy terms (BRESLER, RADCLIFFE and BERRY 1957). In order for the relation to have general significance, it is also clear that body size must be taken into account.

Figure 1 shows the relation between $\dot{E}_w$ and v^2. The present author's results are shown in large solid circles (males, mean values), large open circles (females, mean values) and small solid circles (males, single values). The results of other investigators (ATZLER and HERBST 1927; BENEDICT and MURSCHHAUSER 1915; BOOYENS and KEATINGE 1957; DANIELS, VANDERBIE and WINSMANN 1953; DURNIN and MIKULICIC 1956; MAHADEVA, PASSMORE and WOOLF 1953) are shown in large solid triangles (males, mean, values), large open triangles (females, mean values) and small solid triangles (males, single values). The short vertical lines represent standard deviations above and below the mean. The standard deviations are of the order of 10% of the mean at speeds of about 100 meters/min and may be substantially less at lower speeds.

For speeds up to about 90 meters/min, the differences between males and females are negligible. The present author found a slightly higher value of $\dot{E}_W$ for females at 97.6 meters/min than for males, but this difference was largely abolished when the respirometer was carried by another person, rather than worn by the subject. Booyens and Keatinge (1957) found significantly lower values of $\dot{E}_W$ for women than for men at speeds of 91 and 107 meters/min, and the equation of Cotes, Meade and Wise (1957) predicts lower values for women than for men, owing to the smaller pace-length in women.

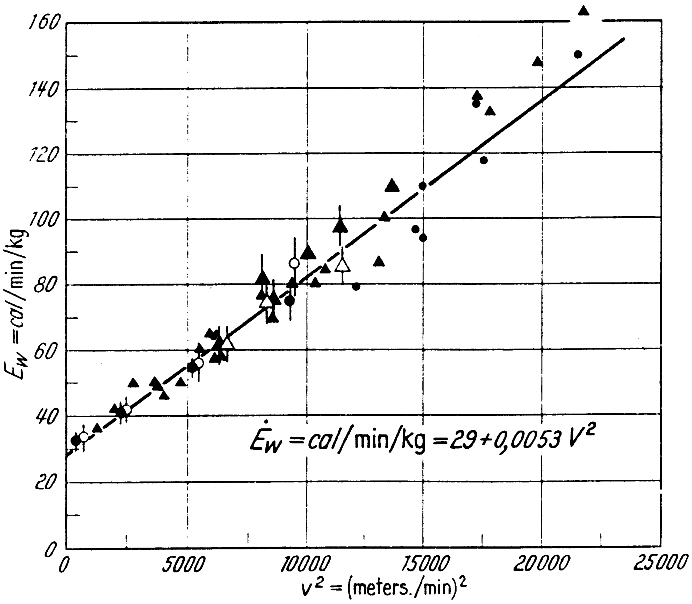

Fig. 1. The linear relation between energy expenditure in cal/min/kg and the square of the speed. Experimental points derived from the studies of a number of different investigators. See text for further description

While it is not the intent of the present author to deny the validity of the finding of Booyens and Keatinge (1957) that $\dot{E}_W$ is smaller in women than in men, a possible source of error in their experiments was the assumption that energy expenditure would be stabilized to within 5% of steady-state values after the subjects had walked 45 meters at speeds of 91 and 107 meters/min. This is an unsafe assumption. The present author found that in 4 cases out of 7, women walking at 97,6 meters/min did not fully stabilize in a period as long as 10 minutes, or 976 meters. It is therefore suggested that Booyens and Keatinge (1957) may have underestimated $\dot{E}_W$ in the female subjects relative to the males.

One of the great advantages in using the Max Planck respirometer rather than the Douglas bag is that the pulmonary ventilation rate can be checked as frequently as desired during an experiment, and thus failure of stabilization noted.

The straight line of Figure 1 has the equation:

$$\dot{E}_W = 29 + 0.0053\, v^2 \tag{1}$$

In spite of the fact that it does not take into account possible sex-differences, and is based upon data derived from walking upon treadmill, floor, outdoor path, and grass, the equation adequately predicts $\dot{E}_w$ for speeds up to 100 meters/min. There is indication of some disorganization of experimental points above that speed, which is to be expected at higher levels of metabolic activity. Even at speeds of 150 meters/min, however, the straight line predicts $\dot{E}_w$ within 10 to 15%.

Table 2. *Predicted values of* $\dot{E}_w$ *from equation (1) and from equation of* COTES, MEADE and WISE (1957), *compared with average experimental values of* BOOYENS and KEATINGE (1957)

	v meters/min	n steps/min	$\dot{E}_w$ experimental cal/min/kg	$\dot{E}_w$ calculated (equation 1)	$\dot{E}_w$ calculated (COTES et al.)
Male	91	106	82	73	74
Female	91	126	75	73	73
Male	107	115	98	90	87
Female	107	141	85	90	83

Table 2 shows the values of $\dot{E}_w$ predicted by equation 1 and by the equation of COTES, MEADE and WISE (1957), together with the experimental values of the average man and woman in the study of BOOYENS and KEATINGE (1957). The physical characteristics of the subjects are taken as: Man: w, 69; d, 0,27; R, 0,91. Woman: w, 57; d, 0.25; R, 0.86.

From equation (1), the value of $\dot{E}_w$ is 29 when $v = 0$. The interesting question arises, what is the meaning of $v = 0$? This question was answered by two sets of experiments in which the value of $\dot{E}_w$ was measured (a) in subjects quietly standing and (b) in subjects walking as slowly as compatible with normal balance. The mean value for $\dot{E}_w$ in 5 subjects was 21.2 standing and 28.6 slowly walking. The intercept-value 29 in equation (1) therefore represents the value of $\dot{E}_w$ when the subject is walking as slowly as possible.

2. The Relation between E_m and v. It has been a frequent practice to calculate energy expenditure in terms of distance walked. In particular, the group of investigators at the Max Planck Institute have made important use of this function (MÜLLER and HETTINGER 1952; MÜLLER and HETTINGER 1953; REITEMEYER 1955). The curve relating energy expended per meter walked per kg to speed is concave upward. The mathematical form of this curve will now be deduced from equation (1):

Dividing equation (1) by v:

$$\frac{\dot{E}_w}{v} = E_m = \frac{29}{v} + 0.0053\,v \tag{2}$$

The curve is a hyperbola and is plotted in Figure 2. As v approaches zero, E_m becomes indefinitely large. As v becomes indefinitely large, E_m also becomes indefinitely large. The curve passes through a minimum, which may be determined by differentiating E_m with respect to v and equating to zero. The result is that E_m is a minimum, having the value 0.78 cal/meter/kg, when v = 74 meters/min. The corresponding $\dot{E}_w$ is 58 cal/min/kg, which represents an $\dot{E}$ of 4 Kcal/min for a 70-kg person.

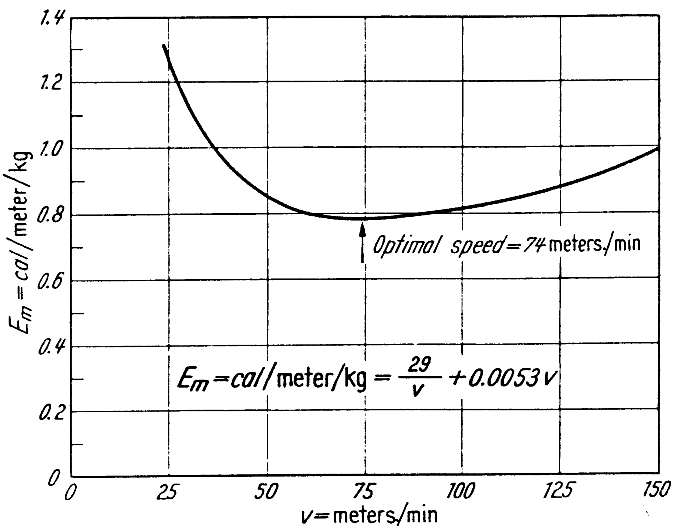

Fig. 2. Relation between energy expenditure in cal/meter/kg and speed, as derived from equation of Figure 1

The curve of Figure 2 is based upon average values. It is almost flat between approximately v = 65 and v = 85 meters/min. In any single subject, the curve, while having the same general characteristics as that of Figure 2, may differ in steepness, as illustrated in Figure 3, which is the curve of a normal male 180 cm tall and weighing 73 kg.

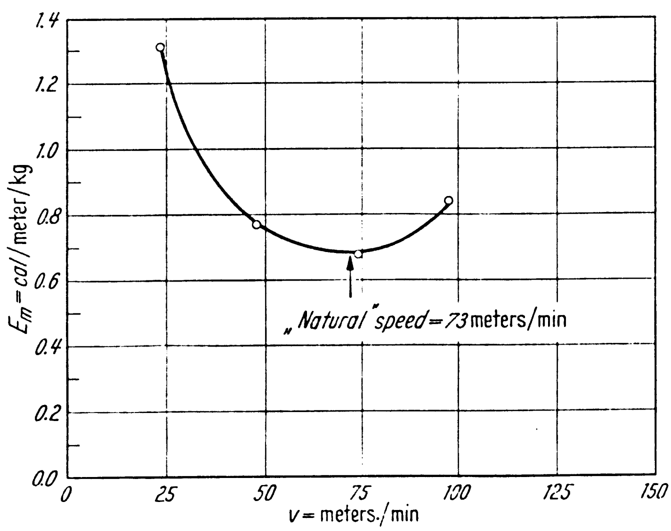

Fig. 3. Normal male subject. Relation between energy expenditure in cal/meter/kg and speed. Arrow indicates the "natural" walking speed of subject

3. The Optimal Speed of walking. If a subject is told to walk at a speed which is „natural" or „comfortable" for him, he adopts a speed which lies at or near the minimum of the E_m curve. For example, the subject of Figure 3 adopted a speed of 73 meters/min. The relation of the „natural" speed to the E_m curve is rather dramatically shown in Figure 4, which shows the E_m curves and the natural speeds of an aboveknee amputee walking with a well-fitted prosthesis and with forearm crutches. The natural speed designated by an arrow, lies close to the minimum of the E_m curve in each case.

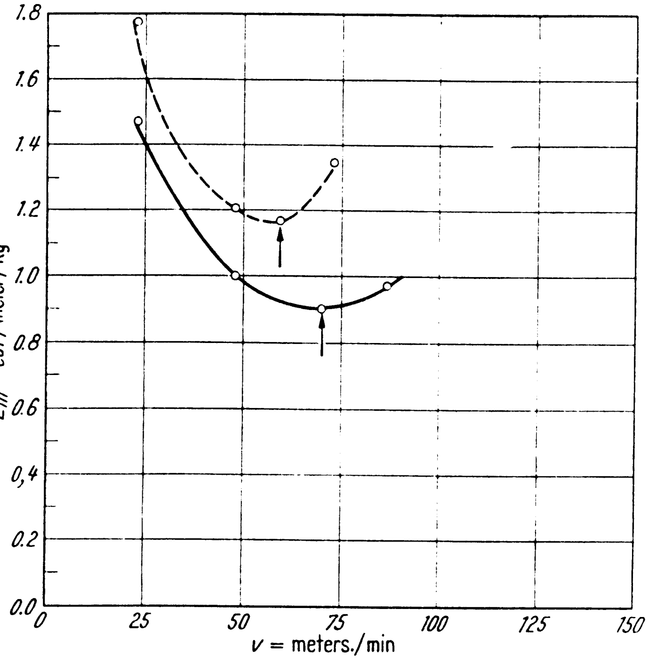

Fig. 4. Male amputee, walking with an above-knee prosthesis only and then with forearm crutches only. Relation between energy expenditure in cal/meter/kg and speed. Arrows indicate "natural" walking speeds. ------ Forearm crutches only; ——— Above-knee prosthesis only; ↑ "Natural" speed

One of the most interesting problems in physiology is to elucidate the built-in mechanism by which a person tends to adopt an optimal walking speed such that energy expenditure per unit distance is a minimum.

Summary and Conclusions

Energy expenditures in 12 male and 7 female untrained adult subjects were measured with Max Planck respirometers while walking in the laboratory at speeds of 24.4, 48.8, 73.2 and 97.6 meters/min. The results were collated with those of other investigators, and the following conclusions were drawn:

1. During level walking, the energy expenditure is a linear function of the square of the speed. The relation is: $\dot{E}_w = 29 + 0.0053\, v^2$, where $\dot{E}_w$ is energy expenditure in cal/min/kg, and v is speed in meters/min.

2. The energy expenditure per unit distance walked is derived from the above equation and is shown to be a hyperbola having a minimal value of 0.78 cal/meter/kg at a speed of 74 meters/min.

3. A given subject adopts a "natural" speed of walking that corresponds to a minimal value of the energy expenditure expressed as cal/meter/kg.

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